A FOURIER ANALYSIS OF THE U.S. DAIRY INDUSTRY

By

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Abstract

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The measurement of economies of scale and scope are extremely valuable for predicting growth and/or product diversification. However, the measurement of these estimates is only useful to the extent the model of the production technology is unbiased and accurate to an appropriate degree. Due to the changing dynamics in the U.S. dairy industry, several economic studies have been conducted to measure economies of scale and scope with the hope of understanding the rapid change in this near perfectly-competitive industry.

This study measures both economies of scale and scope using data from the National ARMS survey and a cost function modeled on the Fourier functional form, which has been shown to provide a global approximation of the unknown function. We find evidence of economies of scale and scope in the dairy industry. We compare our estimates to estimates using other functional forms, including the translog.

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1. INTRODUCTION

The estimation of cost economies has proven invaluable for industry and policy makers alike. Measurements of returns to scale for example, are often used to explain why larger firms enjoy a higher degree of cost reduction. However, these measurements are only useful if they are estimated from an unbiased, accurate approximation of the underlying (unknown) production technology. Since their introduction in the 1970's, the so-called Diewert-flexible (or locally-flexible) functional forms, such as the translog, have been widely used to calculate these measurements despite their well-documented limitations. In fact, Huang and Wang (2004) and Wheelock and Wilson (2001) document that using the translog yields unreliable estimates of economies of scale in the banking industry. To determine the extent of imprecision of the translog, this paper will develop cost economy estimates using both the translog and the globally-flexible Fourier functional form proposed by Gallant (1981, 1982). The semi non-parametric form of the Fourier leads to a more complicated and cumbersome estimation process, but it allows for more reliable estimates of our desired measurements because of its global flexibility.

We chose the U.S. Dairy industry for analysis due to the dramatic changes it has recently experienced and because economies of scope and scale have been estimated for the industry by other methods. The number of dairy firms has decreased dramatically. Both Nonparametric and parametric research has been conducted to test for economies of scale and scope, which are viewed as possible explanations for the decrease in firm numbers as firms consolidate to take advantage of these economies. Previous estimates indicate the presence of both economies of scale and scope for the dairy industry, and this study will use a semi non-parametric approach to determine whether those findings are corroborated using the translog and Fourier functional forms.

This paper will estimate translog and Fourier cost functions in order to measure scale and scope economies for the dairy industry using data from the 2000 ARMS Phase III Survey of dairy farms. Four cost economy measures will be calculated – overall scale economies, ray scale economies, scope economies, and within-sample scope economies.

The Fourier flexible form has been used extensively in the banking industry (e.g., Huang and Wang (2004), Mitchell and Onvural (1996), Kasman (2002), Wheelock and Wilson (2001), but, except for the early work done by Chalfant and Gallant (1985), it has seen little use in studies of U.S. agriculture. This is perhaps due to the technical difficulties associated with its construction and estimation (Huang and Wang 2004) and to the need for a large number of firm-level observations. Using firm-level data from the ARMS Survey, we hope to shed light on scale and scope economies in the dairy industry with this ideal functional form.

Following a short, non-technical introduction to the Fourier series in Section 2, we explain how the model is constructed in Section 3, along with the estimation procedure and a definition of the scale and scope estimates we plan to measure. We describe the data in Section 4, report results in Section 5, and conclude in Section 6.

2. FOURIER COST FUNCTION

While the translog, Box-Cox, and other Diewert-flexible functional forms have been used frequently to measure economies of scale and scope, there are in fact several well-documented problems with these forms that prevent accurate, unbiased estimation. To understand why this is the case, it is essential to recognize that the translog functional from is merely a second-order Taylor series expansion, ignoring the higher order terms. It is merely a local approximation of an unknown function about a point, typically the sample mean. This results in parameter estimates which are globally inconsistent with the Taylor expansion of the unknown function. To illustrate why this is the case, consider the diagram from White (1980) in Figure 1.

In Figure 1, $g(Z_i)$ represents the unknown function to be estimated, $T(Z_i)$ is the Taylor series approximation of $g(Z_i)$ at the data means, and $L(Z_i)$ is the ordinary least squares estimate based on the regression of Y_i on Z (White 1980). It can be seen in Figure 1 that $T(Z_i)$ and $L(Z_i)$ have different slopes and intercepts, and in fact, most of the observations (represented above by dots) lie below the Taylor series approximation.

In the early 1980s, Gallant provided one solution to the problem raised by White and others by using a Fourier series to produce a global approximation of the unknown function. A Fourier series is an expansion of a periodic function in terms of an infinite sum of sines and cosines (Weisstein 2009):

(1) $$f(x) = \frac{1}{2}u_0 + \sum_{h=1}^{\infty}[u_h \cos(x)] + \sum_{h=1}^{\infty}[v_h \cos(x)]$$

where u_h and v_h are parameters to be estimated or solved for depending on the application, x is the independent variable, cos is cosine, and sin is sine. In reality, any two mutually orthogonal polynomials could be used in the infinite sum, but sine and cosine are used most frequently for simplicity. Using a Fourier series is an excellent way to break up an arbitrary unknown function into a collection of sums that can be approximated to a "practical" level of accuracy (Weisstein). The practical level of accuracy suggested by Gallant is the Sobolev norm, which is the level that will be used for this study (Gallant 1982). For a visual representation of a Fourier series, consider Figure 2. In Figure 2, the black lines indicate the function to be approximated, and the lighter lines indicate Fourier series for increasing orders of approximation. The light red line indicates the first order, the yellow indicates the second order, green is the third order, and blue indicates the fourth order. As the order increases, the approximation becomes closer to the unknown function. In the next section we explain how this series is used to create an econometric model that provides an excellent fit of the underlying unknown cost function.

3. METHODOLOGY

In this section, we provide a brief, non-technical description of how the Fourier series is used to represent a cost function in an econometric model, and then briefly explain the estimation procedure and tests used to examine scale and scope economies.

As noted in Figure 2, the ability to represent an unknown function with a Fourier series depends on the number of trigonometric terms used in the approximation. Depending on the function, sometimes an infinite number of trigonometric terms are needed to represent a function exactly. Data limitations force the researcher to use a truncated Fourier series, incorporating only the trigonometric polynomials that seem most appropriate.

Gallant (1981) proved that a truncated Fourier series can better achieve a specified approximation when it includes a second-order polynomial. Details of that proof are omitted here in the interest of space, and the interested reader can consult Gallant (1981) for more detail. Researchers utilizing the Fourier series typically express this second-order polynomial in natural log form, or as it is now known, the translog function. This is especially convenient for the sake of comparison since the translog model is nested within the Fourier model.

3.1 Model

Following Gallant (1982), the Fourier flexible cost function is specified in the following manner:

(2) $$LnC = u_o + \mathbf{b}'\mathbf{x} + 0.5\mathbf{x}'A\mathbf{x} + \sum_{h=1}^{H}[u_h \cos(\mathbf{k}'_h \mathbf{x}) + v_h \sin(\mathbf{k}'_h \mathbf{x})] + \varepsilon$$

The translog cost function is simply the Fourier flexible cost function without the sum of trigonometric terms, specifically:

(3) $$LnC = u_o + \mathbf{b}'x + 0.5\mathbf{x}'A\mathbf{x} + \varepsilon$$

where LnC is the natural log of total cost; u_o is a constant term to be estimated; $\mathbf{b} = [b_{l1},...b_{lN}, b_{z1},...b_{zM}]$ is an N+M vector of coefficients to be estimated; N is the number of input prices; M is the number of output quantities used in the estimation (for this study, N=3, M=4); the independent variable, $\mathbf{x} = [\mathbf{l}', \mathbf{z}']$, is an N+M vector of $\mathbf{l}$ scaled, natural log, input prices and $\mathbf{z}$ scaled, natural log, output quantities; $\mathbf{a} = [a_{ij}]$ is an (N+M) x (N+M) square symmetric matrix of coefficients to be estimated; u_h, v_h are coefficients to be estimated; and $\mathbf{k}_h = [k_{hl1},...k_{hlN}, k_{hz1},...k_{hzM}]$ is an N+M integer vector which indicates which trigonometric terms to include in the Fourier cost function (the *l* and *z* subscripts are used to differentiate between $\mathbf{k}_h$ components that indicate input prices and output quantities, respectively). Explanations of the variables used in equations (2) and (3) are summarized in Table 1 below.

The $\mathbf{k}_h$ vector and the selection of the *H* parameter is what sets the Fourier cost function apart from the translog. It would be ideal to select $\mathbf{k}_h$ and *H* to allow for the inclusion of all eligible trigonometric combinations, but this is prevented due to a finite sample size. The finite sample size also controls the value for *H*, which is equal to $N^{2/3}$, where *N* is the sample size. Setting a value for *H* in this manner is the result of research from Chalfant and Gallant (1985) and Eastwood and Gallant (1991) who proved that this method allows for consistent and asymptotically normal parameter estimates. Huang and Wang (2004b) further noted that the $\mathbf{k}_h$ vector has to meet the following three stipulations: (1) $\mathbf{k}_h$ cannot be a zero vector; (2) its elements cannot have a common integer divisor; and (3) they must be arranged into a sequence such that their lengths are non-decreasing. Implementing these procedures, our selection of $\mathbf{k}_h$ and H yields the following sequence of trigonometric terms:

(4)
$$
\begin{aligned}
&LnC = u_o + \mathbf{b'x} + (0.5)\mathbf{x'Ax} \\
&+ \sum_{h=1}^{3}[u_h \cos(z_i) + v_h \sin(z_i)] + \sum_{h=4}^{6}[u_h \cos(l_m) + v_h \sin(l_m)] \\
&+ \sum_{h=7}^{9}[u_h \cos(z_i + z_j) + v_h \sin(z_i + z_j)] \\
&+ \sum_{h=10}^{12}[u_h \cos(z_i - z_j) + v_h \sin(z_i - z_j)] \\
&+ \sum_{h=13}^{15}[u_h \cos(l_m + l_n) + v_h \sin(l_m + l_n)] \\
&+ \sum_{h=16}^{18}[u_h \cos(l_m - l_n) + v_h \sin(l_m - l_n)] \\
&+ \sum_{h=19}^{27}[u_h \cos(l_m - l_n + z_i) + v_h \sin(l_m - l_n + z_i)] \\
&+ \sum_{h=28}^{36}[u_h \cos(l_m - l_n - z_i) + v_h \sin(l_m - l_n - z_i)]
\end{aligned}
$$

$$i = 1, 2, 3;\ j = 1, 2, 3; i \neq j; m = 1, 2, 3; n = 1, 2, 3; m \neq n$$

To satisfy theoretical expectations for a cost minimizing firm, a cost function is linear homogeneous, concave, and monotonic in input prices. Linear homogeneity restrictions on the input price variables are satisfied by normalizing the three input prices by the price of a fourth input, which serves as our numeraire. Although we do not impose concavity or monotonicity restrictions, they would not affect the ability of the Fourier flexible form to approximate a function that satisfies them (Gallant 1982). We also maintain symmetry restrictions on share equation parameters, which are implied by a twice continuously differentiable cost function.

To implement the Fourier functional form, it is necessary to scale the l and z variables such that they exist within an interval of 0 to 2π. This corresponds directly to the period length of the orthogonal trigonometric polynomials used in the cost function, sine and cosine. The scaling procedure used in this analysis was first suggested by Gallant (1982), and is outlined here for clarity.

Step 1: Identify the minimum and maximum values of the i^{th} input prices and output quantities, i = 1, 2, 3. Denote these values $p_i^{\min}, p_i^{\max}$ and $y_i^{\min}, y_i^{\max}$ for input prices and output quantities, respectively.

Step 2: Define w_{pi} and w_{yi} such that:

$$
\begin{aligned}
w_{pi} &= 0.00001 - Lnp_i^{\min} \\
w_{yi} &= 0.00001 - Lny_i^{\min}
\end{aligned}
\tag{5}
$$

Step 3: Define M, λ, and μ_i for i =1, 2, 3 (note that Lnp_i^{max} is the largest price across all inputs) such that:

$$
\begin{aligned}
M &= Lnp_i^{\max} + w_{pi} \\
\lambda &= 6 / M \\
\mu_i &= 6 / [Lny_i^{\max} + w_{yi}]\lambda
\end{aligned}
\tag{6}
$$

Step 4: Finally, define vectors $\mathbf{l}_i$ and $\mathbf{z}_i$ for i = 1, 2, 3 (which are composed to create the independent variable vector $\mathbf{x} = [\mathbf{l}, \mathbf{z}]$ in Equation (4)) as follows:

$$
\begin{aligned}
l_i &= (Lnp_i + w_{pi})\lambda \\
z_i &= (Lny_i + w_{yi})\mu_i\lambda
\end{aligned}
\tag{7}
$$

Summary statistics of the above variables in their scaled form are reported in Table 2.

3.1 Share Equations

To increase the efficiency of the estimation, we estimate the Fourier cost equation jointly with its input share equations as an iterative seemingly unrelated system, where the input share is the ratio of expenditures on the input to total expenditures. This approach increases efficiency because of the added restrictions placed on the independent variables. To avoid a singular covariance matrix, one of the share equations will be dropped in the estimation. By using iterative seemingly unrelated regression estimation, the estimated parameters are invariant to the equation deleted. Using Shepard's Lemma, we can recover the share equation as the derivative of ln $C(p,y)$ with respect to ln p_i. We define the input share equation s_i for input i as:

(8) $$s_i = \frac{\partial \ln C(p,y)}{\partial \ln p_i}$$

Using the Fourier functional form, the share equations for $i = 1, 2, 3$ are:

(9) $$s_i = \lambda\{b_{li} + \sum_{j=1}^{3} a_{ij} l_j + \sum_{j=4}^{6} a_{ij} z_{j-3} + \sum_{h=1}^{H} [-u_h k_{hli} \sin(\mathbf{k}'_h \mathbf{x}) + v_h k_{hli} \cos(\mathbf{k}'_h \mathbf{x})]\}$$

where the variables, parameters, and subscripts in Equation 9 are the same as in Equation 4. Note that the share equations for the translog are identical to the share equations of the Fourier model except for the trigonometric terms.

3.3 Scale and Scope Economies

Overall scale economies (OSE) are defined in accordance with Baumol, Panzar, and Willig (1982) as follows. This measure has also been referred to as multi-output scale economies, and it accounts for both product-specific scale economies and scope economies.

(10) $$OSE = \frac{C(p,y)}{\sum_{i=1}^{3} y_i C_i(p,y)}$$

Returns to scale are decreasing, constant, or increasing for *OSE* values of less than, equal to, or greater than one respectively.

To calculate economies of scope, we follow Baumol, Panzar, and Willig (1982):

(12) $$SCOPE = \frac{[C(z_1^m) + C(z_2^m) + C(z_3^m) - C(z_1^m, z_2^m, z_3^m)]}{C(z_1^m, z_2^m, z_3^m)}$$

where z_i^m is the sample mean of output z_i. *SCOPE* measures the percentage in cost saving by producing outputs jointly by the same firm rather than by separate firms. Economies of scope exist for the industry when SCOPE > 0, and diseconomies exist when SCOPE < 0.

4. DATA

For data on output quantities (to create the scaled vector ***z***), we used farm-level data from the 2000 Agricultural Resource Management Survey-Phase III (ARMS) for dairy farms. Our sample consists of 870 dairy farms in 22 dairy-producing states, which represents about 1% of U.S. dairy farms in operation in the year 2000. Farms were grouped within each state by size and commodity. Sampling weights were used in the estimation procedure to adjust for the different sampling weights of groups in the data (Melhim 2009). We obtained input prices (to create the scaled vector ***l***) and output prices from the Economic Research Service (ERS) (Ball, Hallahan, and Nehring 2004).

We aggregated outputs into three major categories – dairy (z_1), livestock (z_2), and crops (z_3) using the farm-level output quantity and state-level output price data. The output aggregation process follows Melhim (2009):

1. "From the ARMS farm-level data, we identified the individual commodities (*i*) which had positive values of production in 2000 in any state (*s*).
2. From the ERS state-level data, we divided the annual state-level receipts plus government payments for each identified commodity by its annual quantity of production, Q_{ist} to compute the state-level annual commodity price, P_{ist}, (including per-unit government payments), for each year (*t*).
3. For each year, we used the computed state-level prices from the previous step to compute a geometric mean U.S. price for each output (*i*), $\overline{P}_{it}$ using each state's share of production quantity, W_{ist} as weights as follows:

$$(13) \qquad \bar{P}_{it} = \prod_{S} \left(P_{ist}\right)^{W_{ist}} \text{ where } W_{ist} = \frac{Q_{ist}}{\sum_{S} Q_{ist}}$$

4. For each year, we used the computed geometric mean prices from the previous step to compute the U.S. geometric mean aggregate price, $\bar{P}_{mt}$ where *m*={dairy, other livestock, crops}. As weights, we used the share of output *i*'s receipts in total U.S. receipts for the aggregate *(m)*, W_{it} as follows:

$$(14) \qquad \text{where } W_{it} = \frac{R_{i\in m,t}}{\sum_{i\in m} R_{it}}$$

5. Finally, for each year, we used the relative differences in the state's output prices, P_{ist} (step 2) from the geometric mean U.S. prices, $\bar{P}_{it}$ (step 3) to calculate the geometric mean state-level aggregate prices, $\bar{P}_{mst}$ using the share of output *i*'s receipts in total receipts for the aggregate *(m)* in state *(s)*, W_{ist} as follows:

$$(15) \qquad \bar{P}_{mst} = \left[\prod_{i\in m} \left(\bar{P}_{ist} / \bar{P}_{it}\right)^{W_{ist}}\right] \bar{P}_{mt} \text{ where } W_{ist} = \frac{R_{i\in m,st}}{\sum_{i\in m} R_{st}} \text{."}$$

The ERS data set provided state-level annual prices for major input categories. However, since the land variable in ARMS is in acres, we calculated a land rental rate per acre instead of using the ERS land price index. First, we calculated the average rental rate for land in Iowa in 2000 by dividing the state's total rent by its total acreage in production. Then, we divided this rate by the ERS land price index for Iowa in 2000. We then multiplied the land price index for all states by that number. This procedure yields an inter-spatially consistent set of land rental prices per acre. The final price index for capital was computed as a geometric mean of the land rental rate and the price of other capital components using expenditure shares as weights. The price indices for material, labor, and non-land capital were already computed as geometric means (Melhim 2009).

After this procedure, we grouped farm-level input quantities into three categories; materials (l_1), labor (l_2) and capital (l_3). The material input included purchased livestock, feed, seed and plant, fertilizer, chemicals, fuel and oil, utilities, and other livestock-related inputs. The labor input category included hired, principal operator, and unpaid family labor. Capital inputs consisted of maintenance and repair, machine hire and custom work, interest, rental and lease payments, depreciation, insurance, and property taxes. We then used state-level prices for the three input groups to derive farm-level implicit input quantities by dividing the farm-level expenses for each input group by its state-level price (Melhim 2009).

5. RESULTS

Both the translog and Fourier models converged using the iterative seemingly unrelated estimation procedure in SAS with system adjusted R-squared values of 0.3982 and 0.8875 respectively. Coefficient estimates are presented in Table A.1 for the translog model and in Table A.2 for the Fourier model. Fourteen of the 27 translog parameter estimates are significant at the 5% level, and 41 of the 99 Fourier parameter estimates are significant at the same level. To determine whether the translog is an adequate specification of the cost function, we computed a Wald test statistic to test whether all the sine and cosine parameters of the Fourier model are jointly zero. With a test statistic of

270.03, we reject this hypothesis at the 5% level of significance (and also at the 1% level). This rejection serves as a rejection of the translog model. Considering the test statistic, the large number of significant Fourier parameters, and its much larger R-square value, we conclude that the Fourier model does indeed provide a closer approximation and more reliable estimation of the unknown cost function. We therefore use the Fourier model as a criterion for judging the extent of error in the translog estimates. Estimates for the overall economies of scale (also referred to as multi-output scale economies) and the economies of scope measures for both the Fourier and translog models can be found in Table 3. All measures are computed at the data means.

The measure of economies of scale suggests that the dairy industry operates in the region of increasing returns to scale at the data means. That result is consistent across functional forms. However, although both functional forms render similar conclusions, only the economies of scale measures obtained from the Fourier are significantly greater than 1.0 at the 5% level. The overall scale economies measure estimated by the translog is numerically only slightly greater than 1.0. In comparison with the Fourier model, the translog model understates the returns to scale, which is counter to the result found by Huang and Wang (2004) for the banking industry.

Our estimates of economies of scope are both much greater than zero and imply increasing returns to diversification. As is the case with the economies of scale measures, the translog function also understates the gains from diversifying output. While both the Fourier and translog estimates are both large numerically, only the Fourier is significantly different from zero at the 5% level. The Fourier estimates are 1.32 and

0.986, respectively for the OSE and SCOPE measures, compared to translog estimates of 1.02 and 0.758. While the Fourier and translog models both imply that the mean dairy farm experiences multi-output economies of scale (OSE) and economies of scope (SCOPE), the much better fit of the Fourier model indicates that the estimated gains from increasing scale and diversifying output are underestimated by using the translog functional form. Using a t-test, we conclude that the Fourier model estimates are significantly different than the estimates produced by the translog model for scale economies but not for scope economies. The associated statistics (computed at the mean) are 3.45 and 0.45. However, the large economies of scope results from both functional forms indicate that firms in the U.S. Dairy industry would be well advised to aggressively consider joint production of outputs to realize and enjoy economies of scope, which will allow for the highest degree of cost savings.

Our conclusion about positive economies of scale and scope for the mean firm in this industry are consistent with Melhim (2009) who used an alternative locally flexible functional form, the normalized quadratic, with the same data as we used. Melhim's (2009) estimate (under certainty) of economies of scope was a statistically significant 0.27, which is also an underestimation when compared to the globally accurate Fourier model. Like the translog, it rendered evidence of statistically insignificant increasing returns to scale. We therefore conclude that both the translog and normalized quadratic functional forms numerically underestimate both scale and scope economies. Our conclusion of increasing returns to scale and positive scope economies is qualitatively the same as Melhim, O'Donoghue, and Shumway's (2009) study which used a nonparametric approach and data from the 1992, 1997, and 2002 US Agricultural Censuses.

In considering alternate firm sizes, we find that scale and scope economies diminish as firm size increases. The Fourier estimates imply that the largest 5% of dairy farms experience constant overall returns to scale and significant and very large scope economies. The translog consistently underestimates the gains to be had from economies of scale and scope even for the largest firms; it implies that they experience significantly decreasing overall returns to scale and insignificant scope economies. We conclude that dairy farms of all sizes would be well advised to continue diversification of production in order to reap the benefits of economies of scope and that the smallest 75% of dairy farms can reap overall economies of scale by additional growth.

6. CONCLUSIONS

Using a Fourier globally flexible functional form, we conclude that the United States dairy industry enjoys both increasing returns to scale and increasing returns to diversification. We conclude that the Fourier model provides a much better fit of the ARMS data than the translog does and that the translog underestimates each economy measure. The significant differences in policy implications and the substantial differences in model fit based on the functional form used should lead more researchers to consider the Fourier flexible form. While in some studies (e.g., Huang and Wang 2004) the translog model overstates the gains to be made from expanding production and/or diversifying and in others it underestimates those gains (e.g., the current study), it is clear that the lack of reliability in estimates from the locally flexible functional form should motivate further attention to the global flexibility of the Fourier form. This same conclusion applies relative to alternative locally flexible functional forms.

Policies geared towards increasing diversification by better access to information could potentially help firms in this industry, as could policies aimed at increasing the scale of production for some, such as the consolidation of smaller firms. However, consolidation warrants careful monitoring due to the potential for adverse environmental consequences (Melhim 2009) and the unlikely but real possibility of adverse impact on the perfectively competitive nature of the dairy industry (Skolrud, O'Donoghue, Shumway, and Melhim 2007).

Further research is warranted in searching for the best basis polynomials for the Fourier series. Sine and cosine are the standard polynomials used for Fourier research in economics, but other orthogonal trigonometric polynomials such as the Jacobi or the Laguerre could perhaps provide a closer approximation than sine and cosine. While it has been shown frequently that the translog functional form is indeed inferior to the Fourier functional form, the use of alternative basis polynomials could improve this newer method even more.

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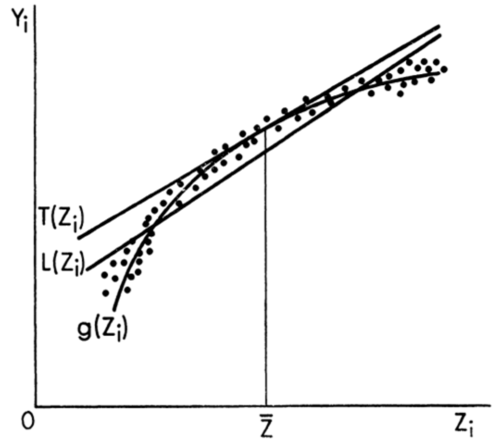
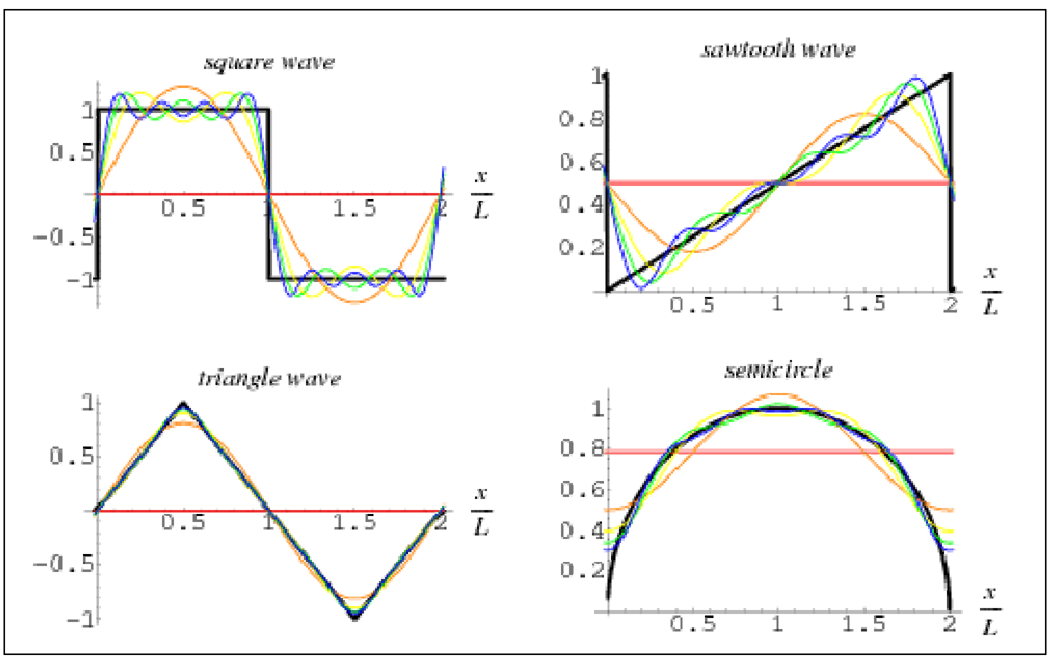
FIGURE 1. Taylor series illustration



SOURCE: White 1980

FIGURE 2. Fourier series illustration



SOURCE: Weisstein 2009.
Order of approximation: red-1, yellow-2, green-3, blue-4

TABLE 1. Variable Definitions for Equations (2) and (3)

Variable/Coefficient	Explanation
LnC	Natural log of total cost
u_o	Constant term to be estimated
$\mathbf{b} = [b_{l1},...b_{lN}, b_{z1},...b_{zM}]$	an N+M vector of coefficients to be estimated, where N is the number of input prices and M is the number of output quantities used in the estimation. For this study, N=3, M=4.
$\mathbf{x} = [\mathbf{l}', \mathbf{z}']$	an N+M vector of **l** scaled natural log-input prices and **z** scaled natural log-output quantities. The derivation of **l** and **z** are described in the text.
$A = [a_{ij}]$	an (N+M) x (N+M) square symmetric matrix of coefficients to be estimated
u_h, v_h	coefficients to be estimated
$\mathbf{k}_h = [k_{hl1},...k_{hlN}, k_{hz1},...k_{hzM}]$	an N+M vector which indicates which trigonometric terms to include in the Fourier Cost Function

TABLE 2. Summary Statistics for Scaled Data

Summary Statistics				
Variable	Mean	Standard Deviation	Minimum	Maximum
Sm	0.389	0.144	0.0368	0.853
Sl	0.290	0.134	0.0404	0.886
Sc	0.243	0.098	0.0073	0.738
l1	1.315	0.812	0.00003	5.253
l2	1.907	1.230	0.00002	5.999
l3	1.175	0.777	0.00003	4.748
z1	3.450	0.767	6.965E-6	6.000
z2	4.721	0.769	2.384E-6	5.999
z3	2.094	2.373	2.608E-6	5.999

TABLE 3. Scale and Scope Results

Scale and Scope Results				
Estimate	Computed at:	Fourier	Translog	$H_0 : B_{Fourier} = B_{Trans\log}$ $H_A : B_{Fourier} \neq B_{Trans\log}$
OSE	Mean	1.32*** (0.09)	1.02 (0.01)	3.45^{R}
OSE	75th Percentile	1.28*** (0.09)	0.95 (0.05)	3.11^{R}
OSE	90th Percentile	1.04 (0.15)	0.89 (0.09)	0.84^{F}
OSE	95th Percentile	1.03 (0.07)	0.74*** (0.08)	2.72^{R}
Estimate	Computed at:	Fourier	Translog	
SCOPE	Mean	0.98*** (0.26)	0.76* (0.44)	0.45^{F}
SCOPE	75th Percentile	0.85*** (0.08)	0.69** (0.31)	0.50^{F}
SCOPE	90th Percentile	0.78*** (0.12)	0.57 (0.42)	0.48^{F}
SCOPE	95th Percentile	0.74*** (0.09)	0.53 (0.57)	0.36^{F}

*** Implies significance at the 1% level, ** implies significance at the 5% level, and * implies significance at the 10% level relative to the null hypothesis of constant returns to scale for OSE and zero scope economies for SCOPE. ^{R} Indicates a rejection of the null hypothesis and ^{F} indicates a failure to reject the null hypothesis of equal economy measures for both functional forms.

APPENDIX

TABLE A.1. Translog Iterated Seemingly Unrelated Parameter Estimates

Parameter[1]	Estimate	Standard Error	t-Value	Pr > $\|t\|$
bl1	-0.046	0.028	-1.62	0.106
bl2	0.895	0.032	27.26	<.0001
bl3	-0.007	0.028	-0.25	0.803
bz1	-0.271	0.279	-0.97	0.332
bz2	0.090	0.244	0.37	0.710
bz3	4.094	0.932	4.39	<.0001
a11	0.014	0.014	0.99	0.321
a12	-0.032	0.005	-6.28	<.0001
a13	0.013	0.018	0.74	0.459
a14	0.126	0.004	26.16	<.0001
a15	0.004	0.005	0.69	0.492
a16	0.00026	0.001	0.19	0.851
a22	-0.0051	0.003	-1.32	0.188
a23	0.031	0.006	4.77	<.0001
a24	-0.145	0.005	-26.03	<.0001
a25	-0.011	0.006	-1.62	0.104
a26	-0.015	0.001	-9.53	<.0001
a33	-0.022	0.024	-0.91	0.364
a34	0.042	0.004	8.70	<.0001
a35	0.007	0.005	1.29	0.197

a36	0.009	0.001	6.72	<.0001
a44	0.502	0.051	9.72	<.0001
a45	-0.053	0.059	-0.89	0.372
a46	-0.066	0.014	-4.72	<.0001
a55	0.099	0.043	2.30	0.021
a56	-0.122	0.011	-10.51	<.0001
a66	-0.048	0.02	-2.43	0.015

[1]Parameters starting with "b" indicate single variables (bl1 is the parameter for l1), and parameters starting with "a" indicate variables multiplied together (a11 is the parameter for l1*l1). Parameters starting with "u" and "v" in Table A.2 indicate variables in the Fourier series (sine and cosine).

TABLE A.2. Fourier Iterated Seemingly Unrelated Parameter Estimates

Parameter[2]	Estimate	Standard Error	t-Value	Pr > $\|t\|$
bl1	-0.024	0.049	-0.500	0.620
bl2	0.288	0.025	11.580	<.0001
bl3	0.091	0.055	1.650	0.099
bz1	-0.282	0.430	-0.660	0.512
bz2	-0.160	0.767	-0.210	0.833
bz3	7.087	1.029	6.890	<.0001
a11	0.014	0.038	0.380	0.706
a12	-0.017	0.004	-3.970	<.0001
a13	0.027	0.011	2.520	0.012
a14	0.050	0.004	13.610	<.0001
a15	0.000	0.003	0.060	0.948
a16	0.000	0.001	-0.090	0.932
a22	0.007	0.007	0.980	0.329
a23	0.045	0.014	3.290	0.001
a24	-0.057	0.003	-17.900	<.0001
a25	-0.004	0.003	-1.450	0.148
a26	-0.005	0.001	-7.680	<.0001
a33	-0.149	0.038	-3.940	<.0001
a34	0.018	0.003	5.830	<.0001

a35	0.002	0.003	1.000	0.320
a36	0.000	0.001	1.020	0.308
a44	0.307	0.133	2.320	0.021
a45	-0.026	0.042	-0.630	0.527
a46	-0.027	0.018	-1.520	0.130
a55	0.165	0.256	0.650	0.517
a56	-0.010	0.012	-0.870	0.385
a66	-0.243	0.026	-9.540	<.0001
u1	0.136	0.394	0.350	0.729
u2	-4.140	1.010	-4.100	<.0001
u3	0.279	0.351	0.790	0.427
u4	0.016	0.024	0.680	0.497
u5	-0.016	0.013	-1.290	0.196
u6	-0.026	0.023	-1.110	0.267
u7	0.117	0.153	0.770	0.442
u8	-0.594	0.255	-2.330	0.020
u9	-0.065	0.086	-0.760	0.446
u10	0.256	0.221	1.160	0.247
u11	0.384	0.267	1.440	0.151
u12	-0.068	0.097	-0.710	0.477
u13	0.020	0.003	5.880	<.0001
u14	-0.006	0.005	-1.360	0.174

u15	-0.016	0.008	-2.070	0.038
u16	-0.006	0.006	-1.200	0.232
u17	-0.101	0.026	-3.870	0.000
u18	-0.038	0.021	-1.790	0.074
u19	0.010	0.003	3.290	0.001
u20	-0.012	0.004	-2.890	0.004
u21	-0.009	0.005	-1.990	0.047
u22	1.120	0.235	4.770	<.0001
u23	0.007	0.006	1.310	0.189
u24	1.095	0.232	4.730	<.0001
u25	0.000	0.002	0.410	0.680
u26	0.007	0.003	2.440	0.015
u27	0.005	0.003	1.900	0.058
u28	0.000	0.002	0.180	0.855
u29	0.007	0.003	2.130	0.034
u30	-0.012	0.004	-3.070	0.002
u31	1.093	0.232	4.720	<.0001
u32	-0.003	0.006	-0.470	0.635
u33	1.137	0.235	4.840	<.0001
u34	0.002	0.002	1.200	0.231
u35	0.001	0.002	0.780	0.434
u36	0.000	0.003	-0.190	0.848

v1	-0.389	0.323	-1.210	0.229
v2	1.496	0.480	3.120	0.002
v3	-0.019	0.152	-0.110	0.768
v4	0.021	0.054	0.390	0.698
v5	0.050	0.014	3.650	0.000
v6	-0.103	0.049	-2.110	0.035
v7	-0.021	0.162	-0.130	0.896
v8	-0.164	0.222	-0.740	0.460
v9	-0.027	0.081	-0.340	0.735
v10	0.008	0.158	0.060	0.956
v11	-0.100	0.176	-0.570	0.568
v12	-0.036	0.078	-0.460	0.647
v13	-0.021	0.005	-3.930	<.0001
v14	0.018	0.005	3.240	0.001
v15	0.036	0.013	2.820	0.005
v16	-0.004	0.008	-0.550	0.580
v17	0.006	0.015	0.400	0.690
v18	0.023	0.013	1.790	0.075
v19	0.007	0.003	2.340	0.019
v20	0.001	0.004	0.340	0.735
v21	0.007	0.004	1.790	0.074
v22	-0.060	0.012	-5.140	<.0001

v23	-0.026	0.007	-3.910	<.0001
v24	-0.063	0.015	-4.230	<.0001
v25	0.001	0.002	0.760	0.445
v26	0.000	0.001	0.140	0.891
v27	0.005	0.003	1.420	0.156
v28	0.004	0.003	1.170	0.241
v29	0.002	0.003	0.470	0.638
v30	-0.006	0.004	-1.650	0.099
v31	0.051	0.013	4.010	<.0001
v32	0.025	0.007	3.760	0.000
v33	0.064	0.015	4.410	<.0001
v34	-0.005	0.002	-2.130	0.034
v35	-0.001	0.003	-0.340	0.736
v36	0.073	.0015	4.182	<.0001